Average and Standard Deviation

Averages and standard deviations can be computed when outcomes and their probabilities are known. The equations for such calculations are below. Note the average return is the arithmetic average, not the geometric or "compounded" average that is typically used when evaluating the time value of money

$$Average\ Return = \sum_{i=1}^{n} Return_i \cdot Probability_i \qquad (1)$$

$$Variance = \left(\sum_{i=1}^{n} Return_i^2 \cdot Probability_i\right) - (Average\ Return)^2 \qquad (2)$$

$$Standard\ Deviation = \sqrt{Variance} \qquad (3)$$

Example:

Flipping a coin is one such way to generate a (random) set of outcomes. Each flip is independent of the previous flip. There are only two possible outcomes—heads or tails—with each outcome having an equal probability of 50%. Say, for instance, that an outcome of heads is associated with a 100% return and an outcome of tails a -50% return. If a large enough sample of coin flips were to occur then the average return from these flips would be

$$Average\ Return = (1.0) \cdot 0.5 + (-0.5) \cdot 0.5$$
$$= 0.5 - 0.25 = 0.25 \text{ or } 25\%$$

The variance could then be computed

$$Variance = (1.0)^2 \cdot 0.5 + (-0.5)^2 \cdot 0.5 - (0.25)^2$$
$$= 0.5 + 0.125 - 0.0625 = 0.5625$$

And standard deviation

$$Standard\ Deviation = \sqrt{0.5625} = 0.75 \text{ or } 75\%$$

Reference

Devore, Jay. Probability and Statistics. Sixth Edition. Brooks/Cole—Thomson Learning. Belmont, CA. 2004. pp. 112-118.